Fuzzy EOQ Model with Permissible Delay in Payments, Multiple Pre-payments using Graded Mean Integration Value Method

Dr. A. THANGAM
Department of Mathematics, Pondicherry University - Community College
Lawspe, Pondicherry, India

Abstract
An inventory model with trade credit financing has several real life applications. The purpose of this paper is to develop an inventory model with fuzzy (imprecise) parameters such as ordering cost, holding cost, interest earned and interest payable rates under multiple pre-payment strategy. For preventing order cancellations, supplier may also allow his retailer to prepay a part of purchasing cost in several installments. In order to encourage sales, the supplier also offers their retailers a delayed payment. Inventory items are not perishable. The retailer's profit function is defuzzified using Graded Mean Integration value method. A solution method is proposed to obtain optimal decision policies. Several managerial insights are obtained with several numerical examples.
Keywords: *Inventory; Fuzzy set; EOQ, Supply chain; Trade credit*

Introduction

Today's global business environment is mostly linked with Permissible delay payments under the situations like Covid-19 Pandemic. The logistic network deals with flow of material items, stock information and financial capabilities of customers. Due to lack of financial flow, permissible delay period offered by supplier is a type of promotional tool to attract his retailer. The revenue generated through sales can be deposited in interest generating account by the retailer and he can also earn interest using deposited amount. Hybrid payment Policy (Taleizadeh et al.[26]) consists of

(a) Multiple prepayment as advance payment to prevent cancellation of the purchase orders and manage the products. To purchase a special product, retailer may have advance payment for setting up orders.
(b) Supplier offers their retailer a delayed payment to stimulate the demand. It increases sales and promotes the product.

Prepayment is preferred by the retailer when their buyers are not giving up their orders. Sometimes customer prefer to pay the purchasing cost in advance in several installments and get the cooperative profit from the manufacturer when the interest earned rate is more than bank's capital rate. (Taleizadeh et al.[26])

Inventory costs, interest rates are not always fixed due to the deficiency in definite form of the information. In global market, the prices of commodities are imprecise due to fluctuations in production and marketing. To handle such ambiguity in the model formulation, a fuzzy set theory (Zadeh [27]) plays a key role. Money flow between nations affects the interest rate. The supply of money within the nation will also affect the interest rate. The interest rate is also affected by the growth rate of nation's economy. The commodity prices, interest earned rate and interest payable rate are not well known. This can be dealt in inventory modelling using fuzzy set theory rather than classical probabilistic theory. With the randomness and vagueness at manufacturing companies, distributers at the warehouse and suppliers at various collection centers, optimal decision policy for a fuzzy inventory control is very effective.

This paper considers a fuzzy model for the supply chain with non-deteriorating items in which supplier is offering hybrid payment policy to his retailer. The inventory cost parameters are considered as fuzzy numbers due to uuncertainty and vagueness in real life business. The purpose of this study is

a) to explore the fuzzy inventory model for optimizing profit in retail industry.
b) to reveal the effects of changes in profit, cycle time with respect to length of credit period.

Literature Review

To face the uncertainty in inventory parameters, fuzzy set theory is used in the following literature. Pertrovic and Sweeney [1] considered the demand rate, lead time and inventory level into triangular fuzzy numbers, and they determined the optimal order quantity with the fuzzy propositions method. Yao et al. [2] developed the Economic Lot Size Production model with customer demand as a fuzzy variable. Yao et al. [3] established a fuzzy inventory system without the backorders in which both the order quantity and the total demand were fuzzified as the triangular fuzzy numbers. Chang [4] created the Economic Order Quantity (EOQ) model with imperfect quality items by applying the fuzzy sets theory, and proposed the model with both a fuzzy defective rate and a fuzzy annual demand. Chang et al. [5] considered the mixture inventory model involving variable lead time with backorders and lost sales. They fuzzified the random lead-time demand to be a fuzzy random variable and the total demand to be the triangular fuzzy number. Based on the centroid method of defuzzification, they derived an estimate of the total cost in the fuzzy sense. Chen *et al*. [6] introduced a fuzzy economic production quantity model with defective products in which they considered a fuzzy opportunity cost, trapezoidal fuzzy cost and quantities in the context of the traditional production inventory model. Maiti [7] developed a multi-item inventory model with stock-dependent demand and two-storage facilities in a fuzzy environment (where purchase cost, investment amount and storehouse capacity are imprecise) under inflation and incorporating the time value of money. Other related articles on this topic can be found in work by Chen and Wang [8], Vujosevic *et al*. [9], Gen et al. [10], Roy and Maiti [11], Ishii and Konno [12], Lee and Yao [13], Yao and Lee [14], Chang et al. [15,16,17], Ouyang et al.[18,19], Yao et al. [20] and other literatures [21,22,23,24,25,26] .

Preliminaries

Here, we use some definitions which are well known in fuzzy set theory.

Definition 1 . A **fuzzy set** F on the given universal set X is set of ordered pairs

$$F = \{(x, \mu_F(x)), x \in X\}$$

$where,\ \ \mu_F : X \rightarrow [0,1]$ is the membership function.

Definition 2. A fuzzy number F is a fuzzy set which satisfies the following conditions:

1. F is normal, that is, **there exists** $x \in R$ such that $\mu_F(x) = 1$
2. $\mu_F(x)$ is piece-wise continuous function.
3. F is convex fuzzy set.

Definition 3. A trapezoidal fuzzy number F =(a,d,c,d) is represented with the following membership function

$$\mu_F(x)=\begin{cases} L(x)=\dfrac{x-a}{b-a} & \text{if} \quad a\le x\le b \\ 1 & \text{if} \quad b\le x\le c \\ R(x)=\dfrac{d-x}{d-c} & \text{if} \quad c\le x\le d \\ 0 \ \text{otherwise} \end{cases}$$

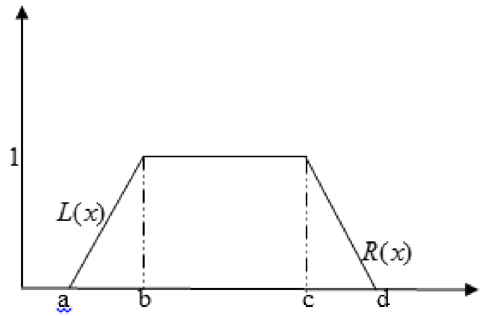

Fig1 : Trapezoidal fuzzy membership function

Arithmetic Operations in fuzzy numbers:

Suppose $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers, then

$$A\oplus B=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$$

$$A\otimes B=(a_1b_1,a_2b_2,a_3b_3,a_4b_4)$$

$$A-B=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1)$$

$$kA=\begin{cases}(ka_1,ka_2,ka_3,ka_4) & \text{if } k>0 \\ (ka_4,ka_3,ka_2,ka_1) & \text{if } k\le 0\end{cases}$$

$$\frac{A}{B}=\left(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1}\right)$$

Graded Mean Integration Method (Chen & Hsieh [28])

The graded mean integration method is used to defuzzify the total fuzzy inventory cost. Consider the trapezoidal fuzzy number $F=(a,b,c,d)$. The defuzzified value of F can be calculated as follows:

$$Z(F)=\left[\int_0^1 \alpha\,\frac{L^{-1}(\alpha)+R^{-1}(\alpha)}{2}\,d\alpha\right]\Big/\int_0^1 \alpha \ d\alpha$$

$$=\frac{a+2b+2c+d}{6}$$

Assumptions and Notations

With the help of following assumptions, the fuzzy inventory model is developed in this paper:

1. Time horizon is infinite.
2. A Supplier-Retailer supply chain with Single item is considered. Items are non-deteriorating with respect to time.
3. Replenishment of items in the inventory is instantaneous.
4. The purchasing cost, selling price, holding cost, interest earned rate and interest payable rates are considered as trapezoidal fuzzy numbers.
5. No shortage is allowed.
6. The Supplier is offering hybrid payment strategy to his retailer.
7. Supplier provides the retailer a fixed permissible delay period M to settle the accounts. If the retailer does not pay at time M, he has to pay penalty at rate I_p.
8. The prepayments by the retailer are payable at multiple equal sizes.
9. Demand rates are assumed to be constant.

Notations:

I(t) – Inventory level at time t
T – Cycle time
Q - Ordering quantity
A – Ordering cost
$\tilde{c}$ – Fuzzy purchasing cost,
$\tilde{s}$ - Fuzzy selling price
$\tilde{h}$ – Fuzzy holding charges per $ per unit item
$\tilde{I}_e$ – Fuzzy Interest rate earned rate per $ per unit item
$\tilde{I}_p$ - Fuzzy Interest payable rate per $ per unit item
λ - Demand rate per annum
M - Retailer's credit period offered by the supplier.
L - Time for advance payments at multiple times
γ - Percentage of purchasing cost prepaid as advance payment.
n - Number of equally spaced prepayments,
TP(T) - Retailer's Total Profit as a function of T.

(A) CRISP MODEL

The annual Total Profit of the supply chain system is TP (T) = Annual Sales Revenue + Annual Interest earned - [Annual Ordering cost+ Annual Purchase cost+Annual Holding cost+ Annual Interest Payable] ------ (1)

The inventory level is described by the following equations

$$I(t)=\begin{cases} Q & \text{if } t=0 \\ Q-\lambda t & \text{if } 0<t<T \\ 0 & \text{if } t=T \end{cases} \quad ---- \quad (2)$$

where Q = λT.

a) Annual sales revenue = $s\lambda$

b) Annual Ordering cost = $\frac{A}{T}$

c) Annual Purchase cost = $c\lambda$

d) Annual holding cost = $\frac{h}{T}\int_o^T I(t)\,dt = \frac{h\lambda T}{2}$

e) Depending upon the values of credit period M, cycle time T and Prepayment time , the following scenarios are considered to calculate Annual Interest Payable and Annual Interest earned:-

Case 1 : if $\frac{M}{\gamma} \le T$

Here, the retailer is offered to prepay γ percentage of purchasing cost in multiple prepayments and $(1-\gamma)$ percentage of purchase cost in trade credit policy. As per the inventory model (Taleizadeh(2020)),

Annual Interest Payable is

$$IP_{1,1} = \gamma c I_k \frac{\lambda T(n+1)L}{2n}$$

Annual Interest earned is

$$IE_{1,1} = (1-\gamma) s I_e \frac{\lambda M}{T}$$

Case 2: if $M \le T \le \frac{M}{\gamma}$

Annual Interest payable due to prepayments

$$IP_{2,1} = \gamma c I_k \frac{\lambda T(n+1)L}{2n}$$

Annual Interest payable due to permissible delay payments

$$IP_{2,2} = c I_k \frac{\lambda (T-M)^2}{2T}$$

Annual Interest earned in case 2

$$IE_{2,1} = (1-\gamma)sI_e \frac{\lambda M}{T}$$

Case 3: if $M \geq T$

Annual Interest payable due to prepayments

$$IP_{3,1} = \gamma c I_k \frac{\lambda T(n+1)L}{2n}$$

Annual Interest earned

$$IE_{3,1} = (1-\gamma)sI_e\lambda$$
$$IE_{3,2} = (1-\gamma)sI_e\lambda(M-T)$$

From the above discussions and Eq.(1), the total profit can be estimated as

$$TP(T) = \begin{cases} TP_1(T) & \text{if } T \geq \dfrac{M}{\gamma} \\ TP_2(T) & \text{if } M \leq T \leq \dfrac{M}{\gamma} \\ TP_3(T) & \text{if } 0 \leq T \leq M \end{cases}$$

-- -- (3)

where

$$TP_1(T) = \left[(s-c)\lambda - \gamma c I_k \lambda \frac{n+1}{n} L + cI_k \lambda M \right] + \frac{1}{2T}\left[2(1-\gamma)sI_e\lambda M - 2A + cI_k\lambda M^2 \right] - \left[h + cI_k \right]\frac{\lambda T}{2}$$

---(3.1)

$$TP_2(T) = \left[(s-c)\lambda - \gamma c I_k \lambda \frac{n+1}{n} L \right] + \frac{1}{T}\left[(1-\gamma)sI_e\lambda M - A \right] - \left[h\frac{\lambda T}{2} \right]$$

----(3.2)

$$TP_3(T) = \left[(s-c)\lambda + sI_e\lambda + (1-\gamma)sI_e\lambda M - \gamma c I_k \lambda \frac{n+1}{n} L \right] - \frac{T}{2}\left[2(1-\gamma)sI_e\lambda + h\lambda \right] - \frac{A}{T}$$

---- (3.3)

(B). Fuzzy model

Here, the crisp inventory parameters are treated as trapezoidal fuzzy numbers.

$\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{s} = (s_1, s_2, s_3, s_4)$,

$h = (h_1, h_2, h_3, h_4)$, $I_e = (Ie_1, Ie_2, Ie_3, Ie_4)$,

$I_p = (Ik_1, Ik_2, Ik_3, Ik_4)$.

$A = (A_1, A_2, A_3, A_4)$

The total profit is

$$TP(T) = \begin{cases} TP_1(T) & \text{if } T \geq \dfrac{M}{\gamma} \\ TP_2(T) & \text{if } M \leq T \leq \dfrac{M}{\gamma} \\ TP_3(T) & \text{if } 0 \leq T \leq M \end{cases}$$

-- (4)

where

$$TP_1(T) = \left[(\tilde{s}\Theta\tilde{c})\lambda \,\Theta \gamma\lambda \frac{n+1}{n} L \;\; c \otimes I_k \oplus \lambda M \;\; \tilde{c} \otimes I_k \right] \oplus \frac{1}{2T}\left[2(1-\gamma)\lambda M \;\; \tilde{s} \otimes I_e \;\Theta 2A + \lambda M^2 \tilde{c} \otimes I_k \right] \Theta \left[h + \tilde{c} \otimes \tilde{I}_k \right]\frac{\lambda T}{2}$$

---(4.1)

$$TP_2(T) = \left[(\tilde{s}\Theta\tilde{c})\lambda \,\Theta \gamma\lambda \frac{n+1}{n} L \;\; c \otimes I_k \right] \oplus \frac{1}{T}\left[(1-\gamma)\lambda M \;\; \tilde{s} \otimes I_e \;\Theta A \right] \Theta \left[h \right]\frac{\lambda T}{2}$$

(4.2)

$$TP_3(T)=\left[\lambda\left(\tilde{s}\Theta\tilde{c}\right)+\lambda\tilde{s}\otimes I_e\oplus(1-\gamma)\lambda M\ \tilde{s}\otimes I_e\Theta\gamma\lambda\frac{n+1}{n}L\tilde{c}\otimes I_k\right]$$
$$\Theta\ \frac{T}{2}\left[2(1-\gamma)\lambda\tilde{s}\otimes I_e\oplus\lambda h\right]\Theta\frac{A}{T}$$

Let

$$\beta=(\beta_1,\beta_2,\beta_3,\beta_4)=[1,2,2,1]$$

Using graded mean integration, the profit function TP(T) is defuzzified into:

$$F\left[TP_1\right]=\frac{\lambda}{6}\left[\sum_{i=1}^{4}\beta_i(s_i-c_{5-i})+\sum_{i=1}^{4}\beta_i(c_iI_{ki}M)+\gamma\frac{n+1}{n}L\sum_{i=1}^{4}\beta_i(c_{5-i}I_{k5-i})\right]$$
$$+\frac{1}{12T}\left[2(1-\gamma)\lambda M\sum_{i=1}^{4}\beta_is_iIe_i+2\sum_{i=1}^{4}\beta_iA_{5-i}+\lambda M^2\sum_{i=1}^{4}\beta_ic_iIk_i\right]$$
$$+\frac{\lambda T}{12}\left[\sum_{i=1}^{4}\beta_i(h_{5-i}+c_{5-i}Ik_{5-i})\right]$$

… (5.1)

$$F\left[TP_2(T)\right]=\frac{\lambda}{6}\left[\sum_{i=1}^{4}\beta_i(s_i-c_{5-i})+\gamma\frac{n+1}{n}L\sum_{i=1}^{4}\beta_ic_{5-i}I_{k5-i}\right]+$$
$$\frac{1}{6T}\left[(1-\gamma)\lambda M\left(\sum_{i=1}^{4}\beta_is_iIe_i\right)+\sum_{i=1}^{4}\beta_iA_{5-i}\right]+\frac{\lambda T}{12}\sum_{i=1}^{4}\beta_ih_{5-i}$$

---- (5.2)

$$F\left[TP_3(T)\right]=\frac{\lambda}{6}\left[\sum_{i=1}^{4}\beta_i(s_i-c_{5-i})+\gamma\frac{n+1}{n}L\sum_{i=1}^{4}\beta_ic_{5-i}I_{k5-i}+\sum_{i=1}^{4}\beta_is_iI_{ei}+(1-\gamma)M\left(\sum_{i=1}^{4}\beta_is_iIe_i\right)\right]$$
$$+\frac{\lambda T}{12}\left[2(1-\gamma\sum_{i=1}^{4}\beta_is_{5-i}I_{e5-i})+\sum_{i=1}^{4}\beta_ih_{5-i}\right]+\frac{1}{6T}\left[\sum_{i=1}^{4}\beta_iA_{5-i}\right]$$

--(5.3)

Let

$$X_1=\frac{\lambda}{6}\left[\sum_{i=1}^{4}\beta_i(s_i-c_{5-i})+\sum_{i=1}^{4}\beta_i(c_iI_{ki}M)+\gamma\frac{n+1}{n}L\sum_{i=1}^{4}\beta_i(c_{5-i}I_{k5-i})\right]$$
$$Y_1=\frac{1}{12}\left[2(1-\gamma)\lambda M\sum_{i=1}^{4}\beta_is_iIe_i+2\sum_{i=1}^{4}\beta_iA_{5-i}+\lambda M^2\sum_{i=1}^{4}\beta_ic_iIk_i\right]$$
$$Z_1=\frac{\lambda}{12}\left[\sum_{i=1}^{4}\beta_i(h_{5-i}+c_{5-i}Ik_{5-i})\right]$$

$$X_2=\frac{\lambda}{6}\left[\sum_{i=1}^{4}\beta_i(s_i-c_{5-i})+\gamma\frac{n+1}{n}L\sum_{i=1}^{4}\beta_ic_{5-i}I_{k5-i}\right]$$
$$Y_2=\frac{1}{6}\left[(1-\gamma)\lambda M\left(\sum_{i=1}^{4}\beta_is_iIe_i\right)+\sum_{i=1}^{4}\beta_iA_{5-i}\right]$$
$$Z_2=\frac{\lambda}{12}\sum_{i=1}^{4}\beta_ih_{5-i}$$

and

$$X_3=\frac{\lambda}{6}\left[\sum_{i=1}^{4}\beta_i(s_i-c_{5-i})+\gamma\frac{n+1}{n}L\sum_{i=1}^{4}\beta_ic_{5-i}I_{k5-i}+\sum_{i=1}^{4}\beta_is_iI_{ei}+(1-\gamma)M\left(\sum_{i=1}^{4}\beta_is_iIe_i\right)\right]$$
$$Y_3=\frac{\lambda}{12}\left[2(1-\gamma\sum_{i=1}^{4}\beta_is_{5-i}I_{e5-i})+\sum_{i=1}^{4}\beta_ih_{5-i}\right]$$
$$Z_3=\frac{1}{6}\left[\sum_{i=1}^{4}\beta_iA_{5-i}\right]$$

Therefore, the defuzzified total profit of the inventory system is

$$F[TP(T)]=\begin{cases}F[TP_1(T)] & \text{if } T\geq\frac{M}{\gamma}\\ F[TP_2(T)] & \text{if } M\leq T\leq\frac{M}{\gamma}\\ F[TP_3(T)] & \text{if } 0\leq T\leq M\end{cases}$$

-- (6)

where

$$F[TP_i(T)]=X_i+Y_iT+\frac{Z_i}{T},\quad i=1,2,3$$

To find the optimum value of cycle time T, the optimum conditions are:

$$\frac{d}{dT}\left(F\left[TP_1\right]\right)=0,\frac{d}{dT}\left(F\left[TP_2\right]\right)=0,\text{and}\frac{d}{dT}\left(F\left[TP_3\right]\right)=0$$

and

$$\frac{d^2}{dT^2}\left(F\left[TP_1\right]\right)<0,\frac{d^2}{dT^2}\left(F\left[TP_2\right]\right)<0,\text{and}\frac{d^2}{dT^2}\left(F\left[TP_3\right]\right)<0$$

The optimal replenishment policies are obtained as below

$$T_1^*=\sqrt{\frac{Z_1}{Y_1}}\ ,\ T_2^*=\sqrt{\frac{Z_2}{Y_2}}\ \text{and}\ T_3^*=\sqrt{\frac{Z_3}{Y_3}}$$

Numerical Example

To illustrate the proposed fuzzy inventory model, we consider the fuzzy inventory parameter values as below:

$$A=(220,225,230,235);\tilde{s}=(10,12,14,16);\tilde{c}=(9,10,11,12);h=(1.6,1.8,2,2.2)$$
$$Ip=(0.12,0.13,0.14,0.15);Ie=(0.18,0.19,0.2,0.21);\gamma=0.11$$

and crisp values M = 0.14; λ = 100. We obtain the optimal solutions as below:

Optimal cycle time = 0.8624 unit time,

Optimal order Quantity = 86 items

Maximum Profit = $ 732

The following is also observed: When credit period time increases the cycle time decreases. When credit period increases the optimal order quantity increases. The retailer wants to buy more items when the credit period is large. When the credit period increases the profit also increases.

Conclusions

In this paper, a fuzzy inventory model with non-deteriorating items is developed when retailer is availing multiple prepayments as well as trade credit policy. Profit function for the Supplier-Retailer supply chain is derived and optimization procedures are also obtained. Several inventory related parameters are fuzzified by assuming triangular fuzzy numbers. Using fuzzy arithmetic operations and graded mean integration method, optimal decision policy for the retailer is explored. Finally numerical example is presented to illustrate mathematical solutions

References

1. Pertrovic, D., Sweeney, E. 1994. "Fuzzy knowledge-based approach to treating uncertainty in inventory control", *Computer Integrated Manufacturing Systems,* 7,, pp. 147-152.
2. Yao, M. J., Chang, P. T. , Huang, S. F., 2005. "On the economic lot scheduling problem with fuzzy demands", *International Journal of Operations Research,* 2, pp 58-71.
3. Yao, J. S. , Chang, S. C., Su J. S., 2000. "Fuzzy inventory without backorder for fuzzy quantity and fuzzy total demand quantity". *Computers and Operations Research*, 27, pp 935-962.
4. Chang, H. C. , 2004. "An application of fuzzy sets theory to the EOQ model with imperfect quality items". *Computers and Operations Research*, 31, pp 2079-2092.
5. Chang, J. S. Yao, and L. Y. Ouyang, 2006. "Fuzzy mixture inventory model involving fuzzy random variable lead-time demand and fuzzy total demand", *European Journal of Operational Research*, 169, pp 65-80.
6. Chen, S. H. , Wang, S. T. , Chang, S. M. , 2005. "Optimization of fuzzy production inventory model with repairable defective products under crisp or fuzzy production Quantity". *International Journal of Operations Research* ,2, pp 31-37.
7. Maiti, M. K., 2008. "Fuzzy inventory model with two warehouses under possibility measure on fuzzy goal". *European Journal of Operational Research*, 188, pp746-774.
8. Chen, S. H., Wang, C. C., "Backorder fuzzy inventory model under functional Principle", *Information Sciences*, 95, pp 71-79.
9. Vujosevic, M.., Petrovic, D. ., Petrovic, R. 1996. "EOQ formula when inventory cost is Fuzzy". *International Journal of Production Economics*, 45, pp 499-504.
10. Gen, M. Tsujimura, Y., Zheng, P. Z., 1997. "An application of fuzzy set theory to inventory control models", *Computers and Industrial Engineering* 33, 553-556.
11. Roy, T. K. Maiti, M., 1997. "A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity", *European Journal of Operational Research*, 99, pp 425-432.
12. Ishii, H., Konno, T., 1998. "A stochastic inventory with fuzzy shortage cost", *European Journal of Operational Research*, 106, pp 90-94.
13. Lee, H. M.. Yao, J. S, 1999. "Economic order quantity in fuzzy sense for inventory without

backorder model", *Fuzzy Sets and Systems*, 105, pp 13-31.

14. Yao, J. S. Lee,H.M., 1999. "Fuzzy inventory with or without backorder for fuzzy order quantity with trapezoid fuzzy number". *Fuzzy Sets and Systems*, 105, , pp. 311-337.
15. Chang, C. T. , Ouyang, L. Y. , Teng, J. T., 2003. "An EOQ model for deteriorating items under supplier credits linked to ordering quantity". *Applied Mathematical Modelling* 27, pp.983-996.
16. Chang, H. C. , Yao, J. S., Ouyang, L. Y., 2004. " Fuzzy mixture inventory model with variable lead-time based on probabilistic fuzzy set and triangular fuzzy number", *Mathematical and Computer Modeling*, 39, pp. 287-304.
17. Chang, P. T., Yao, M. J. , Huang, S. F. , Chen, C. T. 2006. "A genetic algorithm for solving a fuzzy economic lot-size scheduling problem", *International Journal of Production Economics*, 102, pp. 265-288.
18. Ouyang, L. Y. , Wu, K. S. Ho, C. H. 2006. "Analysis of optimal vendor-buyer integrated inventory policy involving defective items", *International Journal of Advanced Manufacturing Technology*, 29, pp.1232-1245.
19. Ouyang, L. Y. , Teng,J.T., Cheng,M.C., 2010. A fuzzy inventory system with deteriorating items under supplier credit linked to order quantity. *Journal of Information Science and Engineering 26*, pp.231-253.
20. Yao, J. S., Huang, W. T. , Huang, T. T. , 2007. "Fuzzy flexibility and product variety in lot-sizing," *Journal of Information Science and Engineering*, 23, , pp. 49-70.
21. Samadi, F., Mirzazadeh, A., Pedram,M.M., 2013. "fuzzy pricing, marketing and service planning in a fuzzy inventory model: A geometric programming approach". *Applied Mathematical Modelling,37,pp. 6683-6694*
22. Gumus,A.T., Guner,A.F., 2009. "A multi-echelon inventory management framework for stochastic and fuzzy supply chains". *Expert Systems with Applications, 36, pp. 5565-5575.*
23. Bera, U.K. Maiti, M.K. Maiti, M., 2012. "Inventory model with fuzzy lead-time and dynamic demand over finite time horizon using a multi-objective genetic algorithm", *Computers & Mathematics with Applications,64, pp. 1822-1838*
24. Mondal,M., Maity,A.K., Maiti,M.K., Manoranjan Maiti 2013. "A production-repairing inventory model with fuzzy rough coefficients under inflation and time value of money". *Applied Mathematical Modelling, 37, pp. 3200-3215*
25. Majumder, P., Bera, U.K. , M. Maiti 2015. "An EPQ Model of Deteriorating Items under Partial Trade Credit Financing and Demand Declining Market in Crisp and fuzzy environment". *Procedia computer Science, 45, pp.780-789.*
26. Taleizadeh, A.A., Tavassoli, S., Bhattacha., A., 2020. Inventory ordering policies for mixed sale of products under inspection policy, multiple prepayment, partial trade credit, payments linked to order quantity and full backordering, *Annals of Operations Research 287, pp 403-437.*
27. Zadeh, L. A. (1965). Fuzzy Sets, *Information and Control, 8, pp 338-353*.
28. Chen, S. H. and C. H. Hsieh, 2000. Representation, Ranking, Distance, and Similarity of L-R Type Fuzzy Number and Application, *Australian Journal of Intelligent Information Processing Systems, 6,pp 217-229*