Bodhi International Journal of
Research in Humanities, Arts and Science
Vol. 10 Special Issue 4 January 2026
E-ISSN: 2456-5571

Self Invertible Graphs with Respect to Finite Groups

T. Gurunathan
Assistant Professor, Department of Mathematics, Sri Kaliswari College (A), Sivakasi

Open Access

Manuscript ID:
BIJ-SPL4-Jan26- MD-009

Subject: Mathematics

Received : 08.08.2025
Accepted : 22.01.2026
Published : 31.01.2026

DOI: 10.64938/bijsi.v10si4.26.Jan009

Abstract
In this paper, we investigate some properties of self invertible graphs of finite groups. We study th algebraic structure of self invrtible graph and find the relation between isomorphic, non-isomorphic groups and their self invertible graphs are presented.

Keywords: Self invertible graph, diameter, S_3, isomorphic graphs

Introduction

Abstract algebra is largely concerned with the study of abstract sets endowed with one or more binary operations. Recently, graphs associated with rings seem to be a more interesting and active area compare to those associated with groups. Our main purpose in this paper is to describe interactions between finite graphs and finite groups, have be exploited to give new result about group-theoretic graphs.

In recent years, a theory of group-theoretic graphs has found many applications in engineering and applied sciences.

Invertible graphs was introduced in 2018 by T. Chalapathi and R.V.M.S.S. Kiran Kumar [1], they defined an undirected simple graph $IG(\Gamma)$ is called invertible graph of a finite group Γ, whose vertex set is Γ, and two vertices a and b in Γ are adjacent in $IG(\Gamma)$ if and only if $a \neq b^{-1}$ or $b \neq a^{-1}$, where a^{-1} is the inverse of a in Γ.

In the present paper, we investigate the graph $SIG(\Gamma)$ of a basic algebraic structure Γ. We classify the finite groups whose self-invertible graphs may be connected or disconnected. Also prove that $SIG(\Gamma)$ is never Eulerian. We also discuss isomorphic theorems with some applications and structure of invertible graphs of finite abelian, non-abelian, and cyclic groups.

Preliminaries

Definition 2.1. [1] Let $(\Gamma, \star)$ be a finite group with identity e. Then an element $a \in \Gamma$ is called a self inverse element of Γ if $a = a^{-1}$, where a^{-1} is the inverse of Γ. The set of self inverse elements of Γ is $S(\Gamma)$ and its cardinality is $|S(\Gamma)|$.

Theorem 2.2. [3] A non-empty subset H of a group Γ is a subgroup of Γ if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.

Theorem 2.3. [1] Let Γ be a cyclic group. Then:
$$|S| = \begin{cases} 1 \text{ if } |\Gamma| \text{ is odd} \\ 2 \text{ if } |\Gamma| \text{ is even} \end{cases}$$

Remark 2.4. [2] If there is a one-to-one mapping $a \leftrightarrow a'$ of the elements of a group Γ onto those of a group Γ', and if $a \leftrightarrow a'$ and $b \leftrightarrow b'$ imply $ab \leftrightarrow a'b'$ then we say that Γ and Γ' are isomorphic and write $\Gamma \cong \Gamma'$. If we put $a' = f(a)$ and $b' = f(b)$ for $a, b \in \Gamma$, then $f : \Gamma \to \Gamma'$ is a bijection satisfying $f(ab) = a'b' = f(a)f(b)$.

Definition 2.5. [1] Let Γ be a finite group. Then an isomorphism from Γ onto Γ is called a group automorphism and set of all automorphisms of Γ is denoted by $Auto(\Gamma)$.

Definition 2.6. [1] An isomorphism from a simple graph G to itself is called graph automorphism of G. Also the set of all graph automorphisms form a group under the operation of composition. This group is also denoted by Auto(G) and is called automorphism group of a graph G.

Self Invertible Graph

Theorem 3.1. The set of all self inverse elements form a subgroup of an abelian group Γ.

Proof. Let $S = \{a \in \Gamma / a^{-1} = a\}$. We know that $e = e^{-1}$, $e \in S$. Therefore $S \neq \emptyset$. Let $a, b \in S$. Then $a^{-1} = a$ and $b^{-1} = b$. Now $(ab^{-1})^{-1} = (b^{-1})^{-1}a^{-1} = ba^{-1} = b^{-1}a = ab^{-1}$. Therefore $a, b \in S \Rightarrow ab^{-1} \in S$. Hence S is a subgroup of Γ.

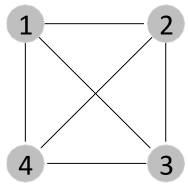
Remark 3.2. More over S is an abelian subgroup of Γ. For, $a, b \in S \Rightarrow ab \in S \Rightarrow (ab)^{-1} = ab$. Now $ab = (ab)^{-1} = b^{-1}a^{-1} = ba \Rightarrow S$ is an ablian subgroup of Γ.

Definition 3.3. Let $(\Gamma, \star)$ be a finite group and $S = \{u \in \Gamma / u = u^{-1}\}$. We define the *self invertible graph* $SIG(\Gamma)$ associated with Γ as the graph whose set of vertices coincides with Γ such that two distinct vertices u and v are adjacent if and only if either $u \star v \in S$ or $v \star u \in S$.

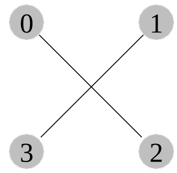
Throughout this paper S will always denote the self invertible elements of the group Γ.

Remark 3.4. Clearly the identity element e is a trivial self invertible element in the finite group Γ. Hence $e \in S$. Consequently the cardinality of S is always greater than or equal to 1.

Example 3.5. Let $\Gamma = V_4$, Klien group of four elements. Then $e = e^{-1}, a = a^{-1}, b = b^{-1}$ and $c = c^{-1}$. Therefore $S = \{e, a, b, c\}$. Then $SIG(\Gamma)$ is the following graph.



Example 3.6. In $(Z_4, \oplus)$, $S = \{0, 2\}$. Then $SIG(\Gamma)$ is the following graph.



Theorem 3.7. For a finite group Γ, the self invertible graph $SIG(\Gamma)$ is complete if and only if $S = \Gamma$.

Proof. $S = \Gamma \Leftrightarrow$ every elememnt of Γ is self inverse $\Leftrightarrow u, v \in \Gamma, uv = (uv)^{-1} \in \Gamma \Leftrightarrow$ any two distinct u, v are adjacent $\Leftrightarrow SIG(\Gamma)$ is complete.

Example 3.8. Let A be any finte set and $\Gamma = (P(A), \triangle)$. Then $S = P(A)$. Therefore by Theorem 3.7 $SIG(\Gamma)$ is complete.

Theorem 3.9. For any non-trivial finite group Γ, the self invertible graph graph $SIG(\Gamma)$ is not empty.

Proof. For any finite group Γ, $e \in S$. Since Γ has more than one element, there exists $a \in \Gamma$, either $a^{-1} = a$ or $a^{-1} \neq a$. If $a^{-1} = a$, then $ae = a \in S$, a

Vol. 10 Special Issue 4 January 2026
E-ISSN: 2456-5571

and e are adjacent. If $a^{-1} \neq a$, then there exist b $\in \Gamma$ such that $a^{-1} = b$. Which implies ab = e. Therefore a and b are adjacent in SIG(Γ). Hence self invertible graph is not empty.

Theorem 3.10. The diameter of a connected self invertible graph SIG(Γ) is either 1 or 2.

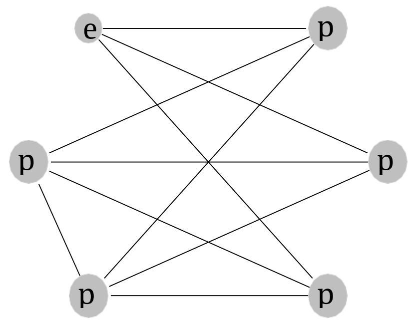
Proof. Let Γ be a finite group with |Γ| > 1. Then consider the following two cases.

Case(i): Suppose S = Γ. In view of Theorem 3.7 SIG(Γ) is complete, and hence *diam*(SIG(Γ)) = 1. Case(ii): Let V (SIG(Γ)) = {e} ∪ S ∪ S′, where S′ is the set of all non invertible elements. Since every element in S is adjacent to e and there is no edge between e and the elements of S′. Now from the construction of SIG(Γ) for every element v ∈ S′, v is not adjacent to e but adjacent to every vertex of S. Therefore *diam*{v} = 2 for each v ∈ S′. Also every element in S is adjacent to both e and the elements of S′, *diam*{u} = 1 for each u ∈ S. Hence *diam*SIG(Γ) = 1 or 2.

Example 3.11. $SIG(S_3)$ is (6,10) graph.

Solution. Since S_3 has 6 elements, $|V(SIG(S_3))| = 6$. The elements of S_3 are $\{e, p_1, p_2, p_3, p_4, p_5\}$, where e= $\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix}$, $p_1 = \begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix}$, $p_2 = \begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}$, $p_3 = \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}$, $p_4 = \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}$, $p_5 = \begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$. Also the elements of S_3 are either odd permutation or even permutation. The set of all odd permutations in S_3 has self inverse. Here p_1, p_2, p_3 are odd permutations. Then $S = \{e, p_1, p_2, p_3\}$. Therefore e is adjacent to p_1, p_2, p_3. Since the product of two odd permutations is an even permutation, p_1, p_2, p_3 are not adjacent to each other. Again the product of two even permutations is an even permutation and p_4, p_5 are not self inverse, $p_4p_5 = e$. Therefore p_4 and p_5 are adjacent. Again product of an odd permutation and an even permutation is an odd permutation, p_1p_4, p_2p_4, p_3p_4, p_1p_5, p_2p_5, p_3p_5 are edges of $SIG(S_3)$. Therefore the total number of edges in $SIG(S_3)$ = 3+1+6 = 10. Hence $SIG(S_3)$ is (6,10) graph.

The following is the graph of $SIG(S_3)$.



Theorem 3.12. Let Γ be a finite cyclic group of even order. Then deg u = $\begin{cases}1 \; if \; u \in S(\Gamma)\\ 2 \; if \; u \notin S(\Gamma)\end{cases}$

Proof. If |Γ| is even, then by Theorem 2.3 |S| = 2. Clearly e ∈ S. Suppose S = {e, a}. Then e and a are adjacent. Therefore the degree of e and a are 1. Let u ∉ S. Then $u \neq u^{-1}$. That is uu≠ e. Since Γ is a group, there exists two elemets v and w such that uv = e and uw = a. Therefore u is adjacent to both v and w. Any other elements in Γ are not adjacent to u. Therefore the degree of u is 2. Hence deg u = $\begin{cases}1 \; if \; u \in S(\Gamma)\\ 2 \; if \; u \notin S(\Gamma)\end{cases}$

Isomorphic Properties of SIG (T)

In this section, we examine isomorphic properties of self invertible graphs of finite groups. Also determine their characteristics. We begin with few examples.

The follpwing Example 4.1 shows that the isomorphic groups and their self invertible graphs are same.

Example 4.1. Consider the two groups $Z_7 - \{0\}$ and $\Gamma = \{T_1, T_2, T_3, T_4, T_5, T_6\}$ where $T_1 = \begin{pmatrix}1 & 2 & 3\,4 & 5 & 6\\ 1 & 2 & 3\,4 & 5 & 6\end{pmatrix}$,

$T_2 = \begin{pmatrix}1 & 2 & 3\,4 & 5 & 6\\ 2 & 4 & 6\,1 & 3 & 5\end{pmatrix}$,

$T_3 = \begin{pmatrix}1 & 2 & 3\,4 & 5 & 6\\ 3 & 6 & 2\,5 & 1 & 4\end{pmatrix}$,

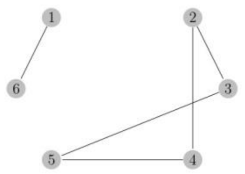
$T_4 = \begin{pmatrix}1 & 2 & 3\,4 & 5 & 6\\ 4 & 1 & 5\,2 & 6 & 3\end{pmatrix}$,

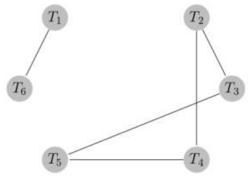
$T_5 = \begin{pmatrix}1 & 2 & 3\,4 & 5 & 6\\ 5 & 3 & 1\,6 & 4 & 2\end{pmatrix}$,

$T_6 = \begin{pmatrix}1 & 2 & 3\,4 & 5 & 6\\ 6 & 5 & 4\,3 & 2 & 1\end{pmatrix}$.

Then $\phi: Z_7 - \{0\} \rightarrow \Gamma$ defined by $\phi(i) = T_i$ is an isomorphism. Their self invertible graphs are

Vol. 10 Special Issue 4 January 2026
E-ISSN: 2456-5571

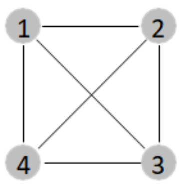


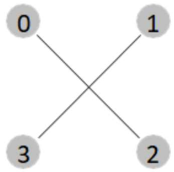


SIG($Z_7 - \{0\}$) SIG(Γ)

The following Example 4.2 shows that grups are not isomorphic and their self invertible graphs are also not isomorphic.

Example 4.2. Consider the groups (Z_4, ⊕) and V_4, Klien group of four elements. Clearly they are not isomorphic. The following are the self inverible graphs.





SIG(V_4) SIG((Z_4, ⊕))

The following Example 4.3 shows that groups are not isomorphic but their self invertible graphs are isomorphic.

Example 4.3. Consider the cyclic group Γ = {I, A, B, C, D, E}, where I $=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$, A= $\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}$, B = $\begin{pmatrix}2 & -2\\ 2 & -2\end{pmatrix}$, C=$\begin{pmatrix}3 & -3\\ 3 & -3\end{pmatrix}$, D = $\begin{pmatrix}4 & -4\\ 4 & -4\end{pmatrix}$, E = $\begin{pmatrix}5 & -5\\ 5 & -5\end{pmatrix}$ with respect to addition modulo 6 and Γ' = {I', A', B', C', D', E'}, where I' $=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$, A'= $\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}$, B' $=\begin{pmatrix}5 & -5\\ 5 & -5\end{pmatrix}$, C'= $\begin{pmatrix}4 & -4\\ 4 & -4\end{pmatrix}$, D'=$\begin{pmatrix}3 & -3\\ 3 & -3\end{pmatrix}$, E' $=\begin{pmatrix}2 & -2\\ 2 & -2\end{pmatrix}$ is an abelian group but not cyclic.

The Cayley table for two groups are

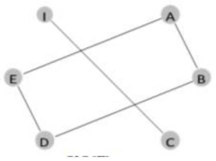
⊕	I	A	B	C	D	E
I	I	A	B	C	D	E
A	A	B	C	D	E	I
B	B	C	D	E	I	A
C	C	D	E	I	A	B
D	D	E	I	A	B	C
E	E	I	A	B	C	D

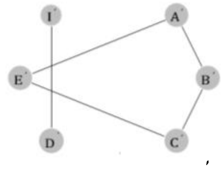
Γ

.	I′	A′	B′	C′	D′	E′
I′	I′	A′	B′	C′	D′	E′
A′	A′	E′	I′	B′	C′	D′
B′	B′	I′	C′	D′	E′	A′
C′	C′	B′	D′	E′	A′	I′
D′	D′	C′	E′	A′	I′	B′
E′	E′	D′	A′	I′	B′	C′

Γ'

The self invertible graphs of Γ and Γ' are





SIG(Γ) SIG(Γ')

The following Theorem 4.5 shows that self invertible graphs of two isomorphic graphs are same but the converse is not true.

Theorem 4.4. Let Γ and Γ′ be finite groups. If Γ ≅ Γ′, then SIG(Γ) ≅ SIG(Γ′). But the converse is not true.

Proof. Suppose Γ ≅ Γ′. Then there is a group isomorphism f from Γ to Γ′ such that f(a) = a′ for every element a in Γ and a' in Γ'. Now define a map ϕ from SIG(Γ) to SIG(Γ') by the relation ϕ(a) = f(a) for every vertex a in Γ. By Remark 2.4, ϕ is a bijection. Now let us prove that ϕ preserves adjacency. For this let ab ∈ S, then ab = e or ab = u for some u. Then f(ab) = f(e) or f(ab) = f(u) ⇒ f(a).f(b) = f(e) or f(a).f(b) = f(u). Since Γ ≅ Γ′, f(e), f(u) ∈ S′. That is ϕ(a).ϕ(b) = e′ or ϕ(a).ϕ(b) = ϕ(u). Therefore ϕ(a) and ϕ(b) are adjacent in SIG(Γ).

Similarly if a is not adjacent to b in SIG(Γ), then ϕ(a) is also not adjacent to ϕ(b) in SIG(Γ). This shows that SIG(Γ) ≅ SIG(Γ). The converse is not true, as the Example 4.2 shows. That is SIG(Γ) ≅ SIG(Γ), it does not necessarily follow that Γ ≅ Γ.

The following result is an analogous result between Auto(Γ) and Auto(SIG(Γ)).

Theorem 4.5. If Γ is a finite group, then Auto(Γ) ⊆ Auto(SIG(Γ)).

Proof. Let ψ ∈ Auto(Γ). Then ψ : Γ → Γ is a group isomorphism. Suppose a and b in Γ are adjacent in SIG(Γ). Then ab ∈ S or ba ∈ S. Let ab = u for some u ∈ S. Now ψ(ab) = ψ(u) or ψ(ba) = ψ(u).Since ψ is an isomprphism, ψ(a)ψ(b) = ψ(u) or ψ(b)ψ(a) = ψ(u). Also if u ∈ S, then ψ(u)

Vol. 10 Special Issue 4 January 2026
E-ISSN: 2456-5571

$\in S$. Therefore $\psi(a)$ is adjacent to $\psi(b)$ in SIG(Γ). This shows that ψ is a group isomorphism of SIG(Γ) to itself. It is clear that $\psi \in$ Auto(SIG(Γ)). Hence Auto(Γ) $\subseteq$ Auto(SIG(Γ)).

The following Example 4.6 shows that the converse of the above Theorem 4.5 is not true.

Example 4.6. Consider the group $Z_5 = \{0, 1, 2, 3, 4\}$ with respect to addition modulo 5. Define a map $\psi : Z_5 \to Z_5$ by $\psi(0) = 0, \psi(1) = 2, \psi(2) = 3, \psi(3) = 4, \psi(4) = 1$. It is clear that Auto($Z_5$) $\subseteq$ Aut(SIG(Z_5)). But $\psi(1 \oplus_5 2) = \psi(3) = 4$ and $\psi(1) \oplus_5 \psi(2) = 2 \oplus 3 = 0$. Therefore $\psi(1 \oplus_5 2) \neq \psi(1) \oplus_5 \psi(2)$ so that ψ is not a homomorphism of Z_5. Then ψ is not an isomorphism.

References

1. T. Chalapathi and R.V.M.S.S. Kiran kumar, Invertible Graphs of Finite Groups, *Computer Sci- ence Journal of Moldova*, Vol.26, no.2(77), 2018.
2. Encyclopedic Dictionary of Mathematicsc: The Mathematical Society of Japan, Vol.1, Kiyosi Ito, Ed., The MIT Press, 2000, 2168p. ISBN-10: 0262590204, ISBN-13:978-0262590204.
3. Jonathan K. Hodge, Steven Schlicker, and Ted Sundstrom, Abstract Algebra:An Inquiry Based Approach (Textbooks in Mathematics), CRC Press, 2014, 595 p. ISBN-13: 978-1466567061, ISBN- 10: 1466567066.
4. Xuan Long Ma, Hua Quan Wei and Guo Zhong, ”The cyclic graph of finite groups”, Algebra, Vol.2013, pp.1-7, 2013.