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Neutrosophic Enclosure of Roots in Newton - Raphson Method

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Abstract

The Newton–Raphson method is one of the most widely used iterative techniques for approximating the roots of nonlinear equations. Classical method lacks an inherent mechanism to capture uncertainty and indeterminacy in the approximation process. In this paper, enclosure of roots in interval analysis is modified into neutrosophic enclosure and newton raphson method is applied. The neutrosophic certified root is found with an explicit quantified indeterminacy component and an excluded falsity region. As the iteration progresses, the indeterminacy measure reduces to zero, with a certified root in neutrosophic enclosure.

Keywords: Neutrosophic random intervals, neutrosophic enclosure, newton–raphson method, neutrosophic certified root, indeterminacy, uncertainty

Introduction

Neutrosophic studies provides the third dimension neutrality which is indeterminacy in nature in addition to possibilities and impossibilities. It is characterized by the triplet (T, I, F) true, indeterminacy and false values which plays vital role in solving real life problems often involving indeterminacy. Classical analysis focuses on the past and present interpretation while neutrosophy is the main focus on the future interpretation. The Newton-Raphson method is a powerful iterative technique to find the roots of the non linearpolynomial equation. The convergence of the root-finding procedure in the traditional Newton-Raphson method begins with an initial guess that may lead to divergence and round-off errors.

In neutrosophic random interval analysis, an enclosure of a root is simply an interval withboundaries containing the true solution of the equation. Interval analysis contains the certified root within the enclosure. But in the modified neutrosophic enclosure the certified roots within the neutrosophic random interval exists with an explicit indeterminacy component, making sure that the certified region takes uncertainty and numerical error into consideration. In this work, the notion of modified enclosure of roots in neutrosophy is applied to the newton–raphson method, where each iteration is represented as a neutrosophic random interval containing the possible root with explicit indeterminacy component. Each iteration progressively reduces the indeterminacy, resulting in both a certified root enclosure, quantified uncertainty and the excluded falsity region.

Preliminaries

Basic definitions like enclosure [1], enclosure of a root [1] and the neutrosophic random intervals [3] were defined with basic operations [3] as follows:

Definition 1 ([1]): Let f be a continuous function from R to R. The interval [a,b] □ R is called a root enclosure of f if there exists $r \epsilon [a, b]$ with $f(r) = 0$. A set X□R is called an enclosure of root of f if $\exists\, r \epsilon X, \ni f(r)$ becomes zero valued.

Definition 2 ([3]): Neutrosophic random numbers is a pool of sets from neutrosophic random intervals.

Definition 3 ([3]): A neutrosophic random interval is defined by [u, v] = $\{l/u \leq l \leq v\}$ where l is a neutrosophic number.

Definition 4 ([3]): Neutrosophic random interval NR(I) is given in the standard form I_1^{NR} = [u, v] where u and v are reals, u < v. when u = v , we get a crisp number [u , u] = u. The neutrosophic random interval can be a subset, not necessarily a crisp number a (closed , open , half-open, half-closed) interval.

Definition 6 ([3]): Operations on neutrosophic random intervals

i) Addition of neutrosophic random intervals
$[u_1, v_1] + [u_2, v_2] = [u_1 + u_2, v_1 + v_2]$ where $u_1 + u_2 < v_1 + v_2$

ii) Subtraction of neutrosophic random intervals
$[u_1, v_1] - [u_2, v_2] = [u_1 - u_2, v_1 - v_2]$ where $u_1 - u_2 < v_1 - v_2$

iii) Division of neutrosophic random intervals
$\frac{I_1^{NR}}{I_2^{NR}} = \left[\frac{u_1}{v_1}, \frac{u_2}{v_2}\right]$ where $\frac{u_1}{v_1} < \frac{u_2}{v_2}$

iv) Multiplication of neutrosophic random intervals
$I_1^{NR} * I_2^{NR} = [u_1 * u_2, v_1 * v_2]$ where $u_1 * u_2 < v_1 * v_2$

Neutrosophic Newton–Raphson Method

In this section, enclosure of roots in interval analysis is modified into neutrosophic enclosure using neutrosophic logic [2]. Neutrosophic enclosure is applied to classical newton–raphson method, where the interval component verifies the existence of the certified root and theindeterminacy component represents uncertainty. The modified definition of neutrosophic enclosure and an algorithm for neutrosophic newton–raphson method is given.

Definition 7 : Let f be a continuous function from R to R. The interval [x_{min} ,x_{max}] □ R is called a neutrosophic enclosure if there exists $\alpha \in$[x_{min} ,x_{max}] + I with $f(\alpha) = 0$, where [x_{min} ,x_{max}] is the deterministic interval and I represents the indeterminacy component.

Algorithm for Neutrosophic Newton–Raphson Method

Step 1: Initial valueNX_0 is obtained from I^{NR} = [$x_{i(min)}$,$x_{i(max)}$] when $f(x_{i(min)}) \cdot f(x_{i(max)}) < 0$ with an indeterminacy component width I_0 and the excluded region falsity F→0 which is beyond the enclosure. The first approximate value of the neutrosophic root is (NX_0, I_0 ,F) .

Step 2: Calculate neutrosophic residual R_i^{NR}= [m,m] with m = f(m_i) where m_i is the mid value of the neutrosophic random interval.

Step 3: Find neutrosophic derivative over D_i^{NR} and $\frac{1}{D_i^{NR}} = \left[\frac{1}{d_{i(\min)}}, \frac{1}{d_{i(max)}}\right]$ where 0 does not belong to D_i^{NR}.

Step4: Estimate N_{1+1}^{NR} from Newton Raphson formula for neutrosophic random interval: $N_{1+1}^{NR} = \left(m_i - R_i^{NR}\left[\frac{1}{D_i^{NR}}\right]\right)$

Step 5: Start the iterative procedure using$NX_{i+1} = NX_i \cap N_{1+1}^{NR}$ with a respective indeterminacy component width I_1 and continue the process.

Step 6: Stop the iteration when I_i→0 and the neutrosophic root (NX_i, I_i, F_i) is computed from the neutrosophic random interval .

Example Problem:

Solve $f(x) = x^3 + x - 1$

Solution : $f(0) . f(1) < 0$

$NX_0 = [0, 1]$ with an indeterminacy component $I_0 \sim 1$

$(NX_0, I_0, F) = ([0,1], 1, 0)$ as $F \rightarrow 0$

First Iteration

Take $m_o = [0.5,0.5]$

Neutrosophic residual R_i^{NR}=[m ,m] with m = f(m$_i$) = $m_i^3 + m_i - 1$

$R_0^{NR} = f([0.5,0.5]) = [0.5,0.5]^3+[0.5,0.5] -1$

$=[0.125,0.125] +[0.5,0.5] -[1,1]$

$= [-0.375,-0.375]$

Neutrosophic Derivative D_i^{NR}over NX_i

$D_0^{NR}= [\ 3x_{min}^2 + 1, 3x_{max}^2 + 1\ ] = [1,4]$

$\frac{1}{D_0^{NR}}= \left[\frac{1}{4}, \frac{1}{1}\right] = [0.25,1]$

Newton Raphson formula for neutrosophic random interval is

$N_{1+1}^{NR} = \left(m_i - R_i^{NR}\left[\frac{1}{D_i^{NR}}\right]\right)$

$N_1^{NR} = \left(m_0 - R_0^{NR}\left[\frac{1}{D_0^{NR}}\right]\right)$

$N_1^{NR} = [0.5,0.5] - [-0.375,-0.375][0.25,1]$

$= [0.5,0.5]+[0.09375,0.375]$

$= [0.59375,0.875]$

$NX_1 = NX_0 \cap N_1^{NR}$

$= [0, 1] \cap [0.59375,0.875] = [0.59375,0.875]$

$NX_1 = NX_0 \cap N_1^{NR}$ with an indeterminacy component$I_1 \sim 0.28125$

$(NX_1, I_1, F)=([0.59375 , 0.875] , 0.28125, 0)$

Second Iteration

$NX_1 = [0.59375,0.875]$

$m_1 = [0.734375,0.734375]$, $f(m_1) = [0.130428314209, 0.130428314209]$

$R_1^{NR}= [0.130428314209 , 0.130428314209]$

$D_1^{NR} = [2.05762, 3.29688]$

$\frac{1}{D_1^{NR}} = [0.303317075, 0.485998386]$

$N_2^{NR} = \left(m_1 - R_1^{NR}\left[\frac{1}{D_1^{NR}}\right]\right)$

$= [0.734375,0.734375] -[0.130428314209, 0.130428314209][0.303317075, 0.485998386]$

$= [0.734375,0.734375] - [0.039561, 0.063388]$

$= [0.694814, 0.670987] = [0.670987,0.694814]$

$NX_2 = NX_1 \cap N2^{NR}$ with an indeterminacy component$I_2 \sim 0.023827$

$(NX_2, I_2, F) =([0.670987, 0.694814] , 0.023827, 0)$

Third Iteration

$NX_2 = [0.670987,0.694814]$

$m_2 = [0.6829,0.6829]$, $f(m_2) = [0.001373, 0.001373]$

$R_2^{NR} = [0.001373, 0.001373]$

$D_2^{NR} = [2.35067, 2.44830]$

$\frac{1}{D_2^{NR}} = [0.42541,0.40845]$

$N_3^{NR} = \left(m_2 - R_2^{NR}\left[\frac{1}{D_2^{NR}}\right]\right)$

$=[0.6829,0.6829] -[0.001373, 0.001373][0.42541,0.40845]$

$=[0.6829,0.6829] -[5.841 \times 10^{-4} , 5.608 \times 10^{-4}]$

$= [0.6823159 , 0.6823392]$

$NX_3 = NX_2 \cap N_3^{NR}$ with an indeterminacy component$I_3 \sim 2.33 \times 10^{-5}$

$(NX_3, I_3, F) = ([0.6823159, 0.6823392] , (2.33 \times 10^{-5}), 0)$

Fourth Iteration

$NX_3 = [0.6823159, 0.6823392]$

$m_3 = [0.68233, 0.68233]$, $f(m_3) = 3.46414 \times 10^{-7}$

$R_3^{NR} = [3.46414 \times 10^{-7}, 3.46414 \times 10^{-7}]$

$D_3^{NR}=[2.396667, 2.396762]$

$\frac{1}{D_3^{NR}} = [0.417246,0.417230] = [0.417230, 0.417246]$

$N_4^{NR} = \left(m_3 - R_3^{NR}\left[\frac{1}{D_3^{NR}}\right]\right)$

$= [0.68233, 0.68233] - [3.46414 \times 10^{-7}, 3.46414 \times 10^{-7}][0.417230, 0.417246]$

$=[0.68233,0.68233] - [1.44534 \times 10^{-7} , 1.4453986 \times 10^{-7}]$

$= [0.682329855, 0.682329855]$

$NX_4 = NX_3 \cap N_4^{NR}$ with an indeterminacy component$I_3 \sim 0$

$(NX_4, I_4, F) = ([0.682329855, 0.682329855], 0,0)$

Results and Discussion

- The method effectively contains a neutrosophic enclosure [0.682329855, 0.682329855] as per the findings.

- The level of indeterminacy component is also reduced $I_1 \sim 0.28125$,$I_2 \sim 0.023827$,$I_3 \sim 2.33 \times 10^{-5}$, $I_3 \sim 0$.
- The neutrosophic certified root is0.682329855.
- This method ensures both existence of certified neutrosophic root, an uncertainty quantification and the excluded region the falsity.
- This modified approach clearly gives a quantified uncertainty explicitly when compared to the classic Newton-Raphson method .
- Higher-order polynomials and system of equations can be solved using this method.

Conclusion

Neutrosophic enclosure structure for the newton–raphson method is introduced by combining classical numerical analysis with neutrosophic logic. This method is useful in numerical procedures with rounding off errors with unavoidable modeling uncertainties. This technique yields the certified enclosures of roots and precise observation of uncertainty. In future study, applications to technological models, nonlinear systems, and optimization challenges where uncertainty is significant will be pursued.

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